The Influence of the Strain Rate on Cutting Processes

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ABSTRACT

From literature on this subject it is known that the process of cutting clay is different from that of water saturated sand. Sand is often modeled as a continuum with an internal friction angle and a sand/steel friction angle but without cohesion and adhesion. Clay is considered to be a continuum with cohesion and adhesion, but with an internal friction angle and a clay/steel friction angle equal to zero. In this paper clay will be considered this way. It has been noticed by many researchers that the cohesion and adhesion of clay increase with an increasing deformation rate. It has also been noticed that the failure mechanism of clay can be of the "flow type" or the "tear type", similar to the mechanisms that occur in steel cutting. Previous researchers, especially Mitchell (1976), have derived equations for the strain rate dependency of the cohesion based on the "rate process theory". However the resulting equations did not allow pure cohesion and adhesion. In many cases the equations derived resulted in a yield stress of zero or minus infinity for a material at rest. Also empirical equations have been derived giving the same problems.

Based on the "rate process theory" with an adapted Boltzman probability distribution, the Mohr-Coulomb failure criteria will be derived in a form containing the influence of the deformation rate on the parameters involved.

KEY WORDS: Clay cutting, Rate process theory, strengthening, dredging.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Boltzman constant (1.3807·10^{-23} J/K)</td>
<td>J/K</td>
</tr>
<tr>
<td>K_1</td>
<td>Grain force on the shear plane</td>
<td>N</td>
</tr>
<tr>
<td>K_2</td>
<td>Grain force on the blade</td>
<td>N</td>
</tr>
<tr>
<td>i</td>
<td>Coefficient</td>
<td>-</td>
</tr>
<tr>
<td>I</td>
<td>Inertial force on the shear plane</td>
<td>N</td>
</tr>
<tr>
<td>N</td>
<td>Avogadro constant (6.02·10^{23} 1/kmol)</td>
<td>-</td>
</tr>
<tr>
<td>N_1</td>
<td>Normal grain force on shear plane</td>
<td>N</td>
</tr>
<tr>
<td>N_2</td>
<td>Normal grain force on blade</td>
<td>N</td>
</tr>
<tr>
<td>p</td>
<td>Probability</td>
<td>-</td>
</tr>
<tr>
<td>R</td>
<td>Universal gas constant (8314 J/kmol/K)</td>
<td>J/kmol/K</td>
</tr>
<tr>
<td>S</td>
<td>Number of bonds per unit area</td>
<td>1/m²</td>
</tr>
<tr>
<td>S_1</td>
<td>Shear force due to internal friction on the shear surface</td>
<td>N</td>
</tr>
<tr>
<td>S_2</td>
<td>Shear force due to soil/steel friction on the blade</td>
<td>N</td>
</tr>
<tr>
<td>T</td>
<td>Absolute temperature</td>
<td>K</td>
</tr>
<tr>
<td>T</td>
<td>Tensile force</td>
<td>N</td>
</tr>
<tr>
<td>v</td>
<td>Cutting velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>W_1</td>
<td>Force resulting from pore under pressure on the shear plane</td>
<td>N</td>
</tr>
<tr>
<td>W_2</td>
<td>Force resulting from pore under pressure on the blade</td>
<td>N</td>
</tr>
<tr>
<td>X</td>
<td>Function</td>
<td>-</td>
</tr>
<tr>
<td>λ</td>
<td>Distance between equilibrium positions</td>
<td>m</td>
</tr>
<tr>
<td>dε/dt</td>
<td>Strain rate</td>
<td>1/s</td>
</tr>
<tr>
<td>dω/dt</td>
<td>Frequency (material property)</td>
<td>1/s</td>
</tr>
<tr>
<td>τ</td>
<td>Shear stress</td>
<td>N/m²</td>
</tr>
<tr>
<td>τ_a</td>
<td>Adhesive shear strength (strain rate dependent)</td>
<td>N/m²</td>
</tr>
<tr>
<td>τ_c</td>
<td>Cohesive shear strength (strain rate dependent)</td>
<td>N/m²</td>
</tr>
<tr>
<td>τ_y</td>
<td>Shear strength (yield stress, material property)</td>
<td>N/m²</td>
</tr>
<tr>
<td>τ_ya</td>
<td>Adhesive shear strength (material property)</td>
<td>N/m²</td>
</tr>
<tr>
<td>τ_yc</td>
<td>Cohesive shear strength (material property)</td>
<td>N/m²</td>
</tr>
</tbody>
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**INTRODUCTION**

Previous researchers, especially Mitchell (1976), have derived equations for the strain rate dependency of the cohesion based on the "rate process theory". However the resulting equations did not allow pure cohesion and adhesion. In many cases the equations derived resulted in a yield stress of zero or minus infinity for a material at rest. Also empirical equations have been derived giving the same problems. Based on the "rate process theory" with an adapted Boltzmann probability distribution, the Mohr-Coulomb failure criteria will be derived in a form containing the influence of the deformation rate on the parameters involved. The equation derived allows a yield stress for a material at rest and does not contradict the existing equations, but confirms measurements of previous researchers. The equation derived can be used for silt and for clay, giving both materials the same physical background. Based on the equilibrium of forces on the chip of soil cut, as derived by Miedema (1987) for soil in general, criteria are formulated to predict the failure mechanism when cutting clay. A third failure mechanism can be distinguished, the "curling type". Combining the equation for the deformation rate dependency of cohesion and adhesion with the derived cutting equations, allows the prediction of the failure mechanism and the cutting forces involved. The theory developed has been verified by using data obtained by Hatamura and Chijiiwa (1975-1977) with respect to the adapted rate process theory and data obtained by Stam (1983) with respect to the cutting forces. However since the theory developed confirms the work carried out by previous researchers its validity has been proven in advance. In this paper simplifications have been applied to allow a clear description of the phenomena involved.

**THE RATE PROCESS THEORY**

It has been noticed by many researchers that the cohesion and adhesion of clay increase with an increasing deformation rate. It has also been noticed that the failure mechanism of clay can be of the "flow type" or the "tear type", similar to the mechanisms that occur in steel cutting. The rate process theory can be used to describe the phenomena occurring in the processes involved. This theory, developed by Glasstone, Laidler and Eyring (1941) for the modeling of absolute reaction rates, has been made applicable to soil mechanics by Mitchell (1976).

![Figure 1: The Boltzman probability distribution.](image)

Although there is no physical evidence of the validity of this theory it has proved valuable for the modeling of many processes such as chemical reactions. The rate process theory, however, does not allow strain rate independent stresses such as real cohesion and adhesion. This connects with the starting point of the rate process theory that the probability of atoms, molecules or particles, termed flow units having a certain thermal vibration energy is in accordance with the Boltzmann distribution (Figure 1):

\[
p(E) = \frac{1}{R \cdot T} \cdot \exp \left( -\frac{E}{R \cdot T} \right)
\]

The movement of flow units participating in a time dependent flow is constrained by energy barriers separating adjacent equilibrium positions. To cross such an energy barrier, a flow unit should have an energy level exceeding a certain activation energy \( E_a \). The probability of a flow unit having an energy level greater than a certain energy level \( E_a \) can be calculated by integrating the Boltzmann distribution from the energy level \( E_a \) to infinity, as depicted in Figure 2, this gives:

\[
p_{E>E_a} = \exp \left( -\frac{E_a}{R \cdot T} \right)
\]

The value of the activation energy \( E_a \) depends on the type of material and the process involved. Since thermal vibrations occur at a frequency given by \( kT/h \), the frequency of activation of crossing energy barriers is:

\[
v = \frac{k \cdot T}{\hbar} \cdot \exp \left( -\frac{E_a}{R \cdot T} \right)
\]

In a material at rest the barriers are crossed with equal frequency in all directions. If however a material is subjected to an external force resulting in directional potentials on the flow units, the barrier height in the direction of the force is reduced by \( (f \cdot \lambda)/2 \) and raised by the same amount in the opposite direction. Where \( f \) represents the force acting on a flow unit and \( \lambda \) represents the distance between two successive equilibrium positions. From this it can be derived that the net frequency of activation in the direction of the force \( f \) is as illustrated in Figure 3:

![Figure 2: The probability of exceeding an energy level \( E_a \).](image)

\[
v = \frac{k \cdot T}{\hbar} \cdot \exp \left( -\frac{E_a}{R \cdot T} \right) \cdot \left\{ \exp \left( \frac{+f \cdot \lambda}{2 \cdot k \cdot T} \right) - \exp \left( \frac{-f \cdot \lambda}{2 \cdot k \cdot T} \right) \right\}
\]
If a shear stress $\tau$ is distributed uniformly along $S$ bonds between flow units per unit area then $f=\frac{\tau}{S}$ and if the strain rate is a function $X$ of the proportion of successful barrier crossings and the displacement per crossing according to $\dot{\varepsilon} = X\cdot|\mathbf{v}|$ then:

$$\varepsilon = 2 \cdot X \cdot \frac{k \cdot T}{h} \cdot \exp\left[\frac{-E_a}{R \cdot T}\right] \cdot \sinh\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right]$$  \hspace{1cm} (5)

with: $R = N \cdot k$

![Figure 3: The probability of net activation in direction of force.](image)

From this equation, simplified equations can be derived to obtain dashpot coefficients for theological models, to obtain functional forms for the influences of different factors on strength and deformation rate, and to study deformation mechanisms in soils. For example:

If $\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right] < 1$ then

$$\sinh\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right] \approx \frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}$$  \hspace{1cm} (6)

resulting in the mathematical description of a Newtonian fluid flow, and:

If $\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right] > 1$ then

$$\sinh\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right] \approx \frac{1}{2} \cdot \exp\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right]$$  \hspace{1cm} (7)

resulting in a description of the Mohr-Coulomb failure criterion for soils as proposed by Mitchel in 1968. Yao and Zeng (1991) used the first simplification (6) to derive a relation between soil shear strength and shear rate and the second simplification (7) to derive a relation between soil-metal friction and sliding speed.

**PROPOSED RATE PROCESS THEORY**

The rate process theory does not allow for shear strength if the deformation rate is zero. This implies that creep will always occur since any material is always exposed to its own weight. This results from the starting point of the rate process theory, the Boltzman distribution of the probability of a flow unit exceeding a certain energy level of thermal vibration.

According to the Boltzman distribution there is always a probability that a flow unit exceeds an energy level, between an energy level of zero and infinity, this is illustrated in Figure 2.

Since the probability of a flow unit having an infinite energy level is infinitely small, the time-span between the occurrences of flow units having an infinite energy level is also infinite, if a finite number of flow units is considered. From this it can be deduced that the probability that the energy level of a finite number of flow units does not exceed a certain limiting energy level in a finite time-span is close to 1. This validates the assumption that for a finite number of flow units in a finite time-span the energy level of a flow unit cannot exceed a certain limiting energy level $E_l$. The resulting adapted Boltzman distribution is illustrated in Figure 4. The Boltzman distribution might be a good approximation for atoms and molecules but for particles consisting of many atoms and/or molecules the distribution according to Figure 4 seems more reasonable, since it has never been noticed that sand grains in a layer of sand at rest, start moving because of their internal energy. In clay some movement of the clay particles seems probable since the clay particles are much smaller than the sand particles. Since particles consist of many atoms, the net vibration energy in any direction will be small, because the atoms vibrate thermally with equal frequency in all directions.

![Figure 4: The adapted Boltzman probability distribution.](image)

If a probability distribution according to Figure 4 is considered, the probability of a particle exceeding a certain activation energy $E_a$ becomes:

$$p_{E_a} = \frac{\exp\left[\frac{-E_a}{R \cdot T}\right]}{1 - \exp\left[\frac{-E_l}{R \cdot T}\right]} \quad \text{if} \quad E_a < E_l$$  \hspace{1cm} (8)

and

$$p_{E_a} = 0 \quad \text{if} \quad E_a > E_l$$  \hspace{1cm} (9)

If the material is now subjected to an external shear stress, four cases can be distinguished with respect to the strain rate.

**Case 1:** The energy level $E_a + \tau \lambda N/2S$ is smaller than the limiting energy level $E_l$ (Figure 5). The strain rate equation is now:

$$\dot{\varepsilon} = 2 \cdot X \cdot \frac{k \cdot T}{h \cdot i} \cdot \exp\left[\frac{-E_a}{R \cdot T}\right] \cdot \sinh\left[\frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T}\right]$$  \hspace{1cm} (10)

with: $i = 1 - \exp\left[\frac{-E_l}{R \cdot T}\right]$

Except for the coefficient $i$, necessary to ensure that the total probability remains 1, equation (10) is identical to equation (5).
Case 2: The activation energy $E_a$ is less than the limiting energy $E_l$, but the energy level $E + \tau \lambda N / 2S$ is greater than the limiting energy level $E_l$ (Figure 6).

The strain rate equation is now:

$$
\dot{\varepsilon} = \frac{X \cdot k \cdot T}{h \cdot i} \left\{ \exp \left[ - \left( \frac{E_a}{R \cdot T} - \frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T} \right) \right] - \exp \left[ - \frac{E_{lT}}{R \cdot T} \right] \right\} \tag{11}
$$

Case 3: The activation energy $E_a$ is greater than the limiting energy $E_l$, but the energy level $E_a - \tau \lambda N / 2S$ is less than the limiting energy level $E_l$ (Figure 7). The strain rate equation is now:

$$
\dot{\varepsilon} = \frac{X \cdot k \cdot T}{h \cdot i} \left\{ \exp \left[ - \left( \frac{E_a}{R \cdot T} - \frac{\tau \cdot \lambda \cdot N}{2 \cdot S \cdot R \cdot T} \right) \right] - \exp \left[ - \frac{E_{lT}}{R \cdot T} \right] \right\} \tag{12}
$$

Equation (12) appears to be identical to equation (11), but the boundary conditions differ.

Case 4: The activation energy $E_a$ is greater than the limiting energy $E_l$ and the energy level $E_a - \tau \lambda N / 2S$ is greater than the limiting energy level $E_l$ (Figure 8). The strain rate will be equal to zero in this case.

$$
\tau = (E_a - E_l) \cdot \frac{2 \cdot S}{\lambda \cdot N} + R \cdot T \cdot \frac{2 \cdot S}{\lambda \cdot N} \cdot \ln \left[ 1 + \frac{\dot{\varepsilon}}{\varepsilon_0} \right] \tag{13}
$$

with: $\varepsilon_0 = \frac{X \cdot k \cdot T}{h \cdot i} \cdot \exp \left[ - \frac{E_{lT}}{R \cdot T} \right]$

According to Mitchell (1976), if no shattering of particles occurs, the relation between the number of bonds $S$ and the effective stress $\sigma_e$ can be described by the following equation:

$$
S = a + b \cdot \sigma_e \tag{14}
$$

Lobanov and Joanknecht (1980) confirmed this relation implicitly for pressures up to 10 bars for clay and paraffin wax. At very high pressures they found an exponential relation that might be caused by internal failure of the particles. For the friction between soil and metal Yao and Zeng (1988) also used equation (14), but for the internal friction Yao and Zeng (1991) used a logarithmic relationship which contradicts Lobanov and Joanknecht and Mitchell, although it can be shown by Taylor series approximation that a logarithmic relation can be transformed into a linear relation for values of the argument of the logarithm close to 1. Since equation (14) contains the effective stress it is necessary that the clay used is fully consolidated. Substituting equation (14) in equation (13) gives:

$$
\tau = a \cdot \left[ (E_a - E_l) \cdot \frac{2}{\lambda \cdot N} + R \cdot T \cdot \frac{2}{\lambda \cdot N} \cdot \ln \left( 1 + \frac{\dot{\varepsilon}}{\varepsilon_0} \right) \right] + b \cdot \left[ (E_a - E_l) \cdot \frac{2}{\lambda \cdot N} + R \cdot T \cdot \frac{2}{\lambda \cdot N} \cdot \ln \left( 1 + \frac{\dot{\varepsilon}}{\varepsilon_0} \right) \right] \cdot \sigma_e \tag{15}
$$

Equation (15) is of the same form as the Mohr-Coulomb failure criterion:

$$
\tau = \tau_c + \sigma_e \cdot \tan(\varphi) \tag{16}
$$
Equation (15), however, allows the strain rate to become zero, which is not possible in the equation derived by Mitchell (1976). The Mitchell equation and also the equations derived by Yao and Zeng (1988-1991) will result in a negative shear strength at small strain rates.

**COMPARISON OF PROPOSED THEORY WITH SOME OTHER THEORIES**

The proposed new theory is in essence similar to the theory developed by Mitchell (1976) which was based on the "rate process theory" as proposed by Eyring (1941). It was no, however, necessary to use simplifications to obtain the equation in a useful form. The following formulation for the shear stress as a function of the strain rate has been derived by Mitchell by simplification of equation (5):

\[
\tau = a \left( \frac{2}{\lambda \cdot N} + R \cdot T \cdot \frac{2}{\lambda \cdot N} \cdot \ln \left[ \left( \frac{\varepsilon}{B} \right) \right] \right) + \frac{b}{X \cdot k \cdot T} \cdot \sigma_e
\]

(17)

with: \( B = \frac{X \cdot k \cdot T}{h} \)

This equation is not valid for very small strain rates, because this would result in a negative shear stress. It should be noted that for very high strain rates the equations (15) and (17) will have exactly the same form. Zeng and Yao (1991) derived the following equation by simplification of equation (5) and by adding some empirical elements:

\[
\ln(\tau) = C_1 \cdot C_2 \cdot \ln(\varepsilon) + C_3 \cdot \ln(1 + C_4 \cdot \sigma_e)
\]

(18)

Rewriting equation (18) in a more explicit form gives:

\[
\tau = \exp \left[ C_1 \cdot \left( \frac{\varepsilon}{e} \right)^2 \cdot (1 + C_4 \cdot \sigma_e) \right] C_3
\]

(19)

Equation (19) is valid for strain rates down to zero, but not for a yield stress. With respect to the strain rate, equation (19) is the equation of a fluid behaving according to the power law named "power law fluids". It should be noted however that equation (19) cannot be derived from equation (5) directly and thus should be considered as an empirical equation. If the coefficient \( C_3 \) equals 1, the relation between shear stress and effective stress is similar to the relation found by Mitchell (1976). For the friction between the soil (clay and loam) and metal Yao and Zeng (1988) derived the following equation by simplification of equation (5):

\[
\tau_b = \tau_{y0} + C_5 \cdot \ln(\varepsilon) + C_6 \cdot \tan [\delta] = \tau_{y0} + C_6 \cdot \tan [\delta]
\]

(20)

Equation (20) allows a yield stress, but does not allow the sliding velocity to become zero. An important conclusion of Yao and Zeng is that pasting soil on the metal surface slightly increases the friction meaning that the friction between soil and metal almost equals the shear strength of the soil.

The above-mentioned researchers based their theories on the rate process theory, other researchers derived empirical equations. Turnage and Freitag (1970) observed that for saturated clays the cone resistance varied with the penetration rate according to:

\[
F = a \cdot v^b
\]

(21)

With values for the exponent ranging from 0.091 to 0.109 Wismer and Luth (1972) confirmed this relation and found a value of 0.100 for the exponent, not only for cone penetration tests but also for the relation between the cutting forces and the cutting velocity when cutting clay with straight blades. Hatamura and Chijiiwa (1975-1977) also confirmed this relation for clay and loam cutting and found an exponent of 0.089.

Soydemir (1977) derived an equation similar to the Mitchell equation. From the data measured by Soydemir a relation according to equation (21) with an exponent of 0.101 can be derived. This confirms both the Mitchell approach and the power law approach.

**VERIFICATION OF THE THEORY DEVELOPED**

The theory developed differs from the other theories mentioned in the previous paragraph, because the resulting equation (15) allows a yield strength (cohesion or adhesion). At a certain consolidation pressure level equation (15) can be simplified to:

\[
\tau = \tau_y + \tau_0 \cdot \ln \left[ 1 + \frac{\varepsilon}{\varepsilon_0} \right]
\]

(22)

If \((d\varepsilon/dt)/(d\varepsilon_0/dt) << 1\), equation (22) can be approximated by:

\[
\tau = \tau_y + \tau_0 \cdot \frac{\varepsilon}{\varepsilon_0}
\]

(23)

This approximation gives the formulation of a Bingham fluid. If the yield strength \( \tau_y \) is zero, equation (23) represents a Newtonian fluid. If \((d\varepsilon/dt)/(d\varepsilon_0/dt) >> 1\), equation (22) can be approximated by:

\[
\tau = \tau_y + \tau_0 \cdot \ln \left[ \frac{\varepsilon}{\varepsilon_0} \right]
\]

(24)

This approximation is similar to equation (17) as derived by Mitchell. If \((d\varepsilon/dt)/(d\varepsilon_0/dt) >> 1\) and \( \tau - \tau_y \ll \tau_y \), equation (22) can be approximated by:

\[
\tau = \tau_y \cdot \left[ \frac{\varepsilon}{\varepsilon_0} \right]^{\gamma/\nu}
\]

(25)

This approximation is similar to equation (21) as found empirically by Wismer and Luth (1972) and many other researchers. The equation (15) derived in this paper, the equation (17) derived by Mitchell and the empirical equation (21) as used by many researchers have been
fitted to data obtained by Hatamura and Chijiiwa (1975-1977). This is illustrated in Figure 9 with a logarithmic horizontal axis. Figure 10 gives an illustration with both axis logarithmic. These figures show that the data obtained by Hatamura and Chijiiwa fit well and that the above described approximations are valid. The values used are \( \tau_y = 28 \text{ kPa}, \tau_0 = 4 \text{ kPa} \) and \( \varepsilon_0 = 0.03 /\text{s} \).

Figure 9: Shear stress as a function of strain rate with the horizontal axis logarithmic.

Figure 10: Shear stress as a function of strain rate with logarithmic axis

It is assumed that adhesion and cohesion can both be modeled according to equation (22). The research carried out by Yao and Zeng (1988) validates the assumption that this is true for adhesion.

In more recent research Kelessidis et al (2007, 2008) utilizes two rheological models, the Herschel-Bulkley model and the Casson model. The Herschel Bulkley model can be described by the following equation:

\[
\tau = \tau_{y,HB} + K \cdot \left( \frac{\varepsilon}{n} \right)^n
\]  

The Casson model can be described with the following equation:

\[
\sqrt{\tau} = \sqrt{\tau_{y,Ca} + \mu_{Ca} \cdot \varepsilon}
\]  

Figure 11 compares these models with the model as derived in this paper. It is clear that for the high strain rates the 3 models give similar results. These high strain rates are relevant for cutting processes in dredging and offshore applications.

DISCUSSION AND CONCLUSIONS

In explaining the failure mechanisms when cutting clay it is assumed that the stresses are constant over the shear plane and over the blade. It has been observed by Hatamura and Chijiiwa (1975-1977) that this might not be true in all cases. The method described in this paper however permits the study of the phenomena occurring during the failure of clay in a simple way. The three possible failure mechanisms can be distinguished and the most important parameters on which the occurrence of a mechanism depends are described.

Figure 12: The horizontal cutting force as a function of layer thickness.

Combining the adapted rate process theory as derived in this paper with the cutting theory as described by Miedema (2010), allows the calculation of the cutting forces as a function of the cutting velocity. Assuming that the cohesion and the adhesion both are in accordance...
with equation (15) and (22) gives an explanation of the fact that the failure mechanism can change when increasing the cutting velocity while keeping all the other parameters involved constant, since the deformation rate in the shear plane will differ from the deformation rate on the blade. This implies that when the cutting velocity changes, there is a change in the ratio between the adhesive force and the cohesive force, so one of the conditions (35) or (36) may be satisfied. This has been noticed in metal cutting. The rate process theory derived is a general theory, not specific to clay. One of the advantages of this is that the rheological behavior of silt and clay can be described by the same equation (15). Equations (23), (24) and (25) prove that equation (22) does not contradict other theories, but these equations validate the use of the theory developed.

The theory in this paper reflects some thoughts of the author in trying to solve the problem of explaining the phenomena causing yield stress.

To really solve this problem, quantum mechanics should be used, but this is beyond the scope of this paper. In using the theory derived in this paper, one should understand that modeling is an attempt to describe phenomena occurring in reality without having any presumption of being reality.

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